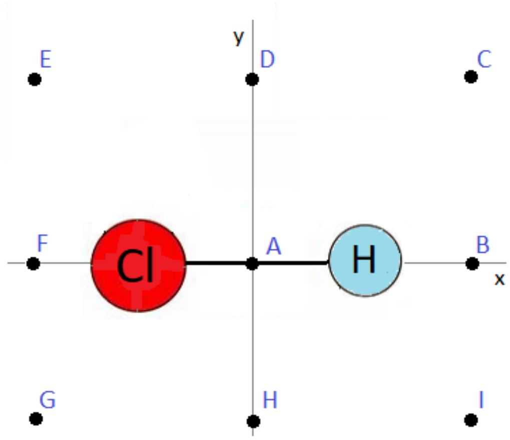
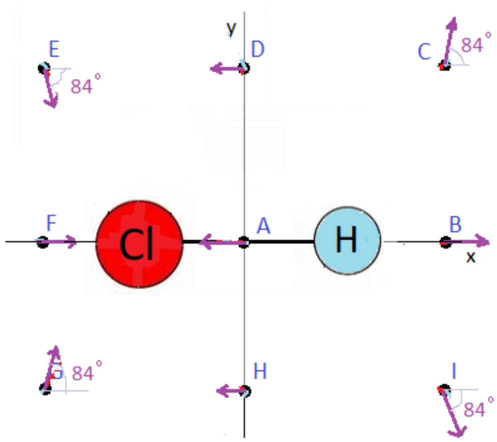
Homework 4 Solutions: Force

Problem 1. Our HCl molecule is back! Recall the Cl atom has got a charge of -e, the H atom a charge of +e, and the bond length is 127pm. Suppose we alternately place a proton and electron at each of those coordinates. What force (magnitude and direction – specified as an angle from the +x axis) would the HCl molecule exert on it? Draw those two forces at each point. You may use the results of our calculations in previous assignments to expedite your work.



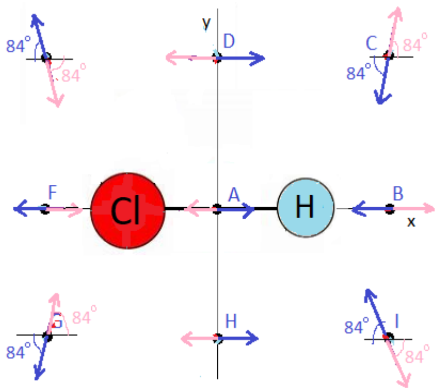
So before we got the electric field to look like this:



with these magnitudes…

$$E_A = 7.14 \times 10^{11} \text{ N/C}$$
$$E_{B,F} = 9.7 \times 10^{11} \text{ N/C}$$
$$E_{D,H} = 1.1 \times 10^{11} \text{ N/C}$$
$$E_{C,E,G,I} = 9.9 \times 10^{10} \text{ N/C}$$

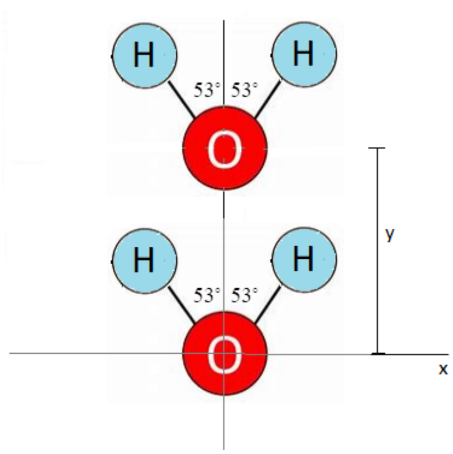
So the forces on a proton (pink) and electron (blue) would look like this:



With these magnitudes:

$$F_A = |q|E_A = (e)(7.14\times10^{11}\text{ N/C}) = 1.14\times10^{-7}\text{ N}$$
$$F_{B,F} = |q|E_{B,F} = (e)(9.7\times10^{11}\text{ N/C}) = 1.55\times10^{-7}\text{ N}$$
$$F_{B,F} = |q|E_{D,H} = (e)(1.1\times10^{11}\text{ N/C}) = 1.76\times10^{-8}\text{ N}$$
$$F_{C,E,G,I} = |q|E_{C,E,G,I} = (e)(9.9\times10^{10}\text{ N/C}) = 1.58\times10^{-8}\text{ N}$$

Problem 2. Let's take another look at our water molecule, recalling that the effective charge of O is -0.70e, and the effective charge of H is +0.35e. This time we're going to place another water molecule, lined up along the same axis. The bottom water molecule will exert a force on the top water molecule. Calculating this force is somewhat annoying because there is a separate force on the top O and each of the top H's. And the net force on the water molecule would be the resultant of these three forces. To avoid that complexity, let's just consider the top O alone. (a) Where is the force on the top O zero? (b) Where is it attractive? (c) Where is it repulsive? You can use the results of calculations in a previous homework assignment to (vastly) expedite your work. (d) Is the second water molecule stable with respect to perturbations? That is to say, does the electric force on it act like a spring so that it will restore the top O to its equilibrium (F = 0) point if it is slightly displaced from the equilibrium point in either direction?



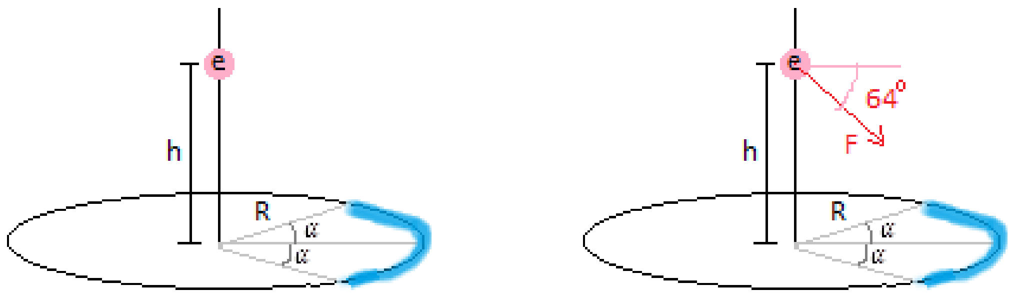
(a) Last time we found that the bottom water molecules electric field was zero at roughly 130pm. So this is where the force on top O will be zero as well.

(b) The bottom water molecules field was positive above 130pm, which would correlate to a negative (i.e. downward, attractive) force on top O since top O is negative.

(c) Similarly, E was negative below 130pm, which would correlate to a positive (i.e. upward, repulsive) force on top O, since top O is negative.

(d) Well, yes. And this is, at a very basic level, how water molecules stick together in the liquid state.

Problem 3. Let's take another look at the charged R = 3m semi-ring from HW 1, where we smeared 7nC over the $\alpha = \pm 30°$ interval. Say we place an electron at the the point along the central axis at h = 6m. What would be the magnitude and direction of the force on it? Display it on the diagram. Might want to use the result from HW 1 to help out.



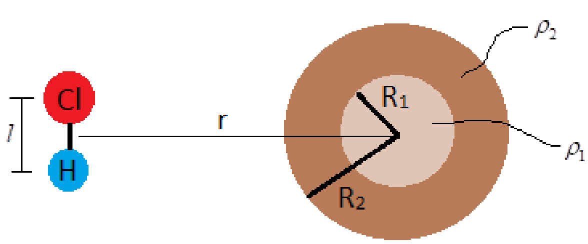
So last time we found that the electric field there was:

$$\mathbf{E} = 1.42\,\text{N/C} \;@\; 64^\circ \text{ above -x axis}$$

And so it follows that:

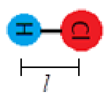
$$\mathbf{F} = (-e)\mathbf{E} = (e)1.42\,\text{N/C} \;@\; 64^\circ \text{ below +x axis}$$
$$= 2.27\times10^{-19}\,\text{N} \;@\; 64^\circ \text{ below +x axis}$$

Problem 4. Let's resurrect our 'typical cylindrical shaped branch'. Remember say R_1 = 5cm, and R_2 = 10cm. And ρ_1 = 7pC/m^3, and ρ_2 = 3pC/m^3. Now let's say that we have an overall neutral molecule, like HCl outside the branch, a distance r = 5m away.



(a) What is the net force on this molecule at present? Don't calculate anything.
It's zero!

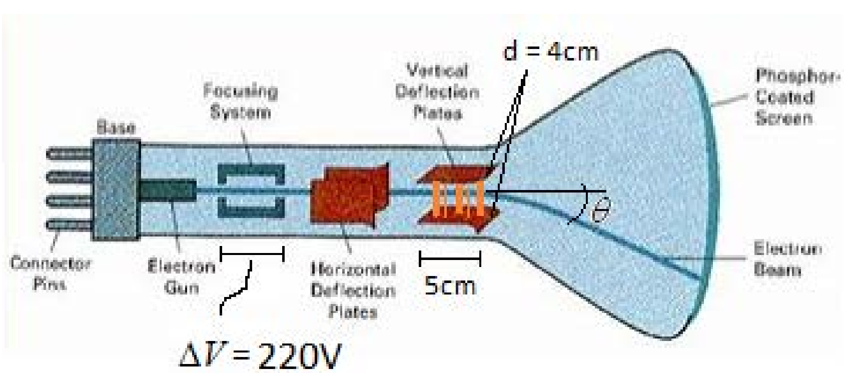
(b) Which way is the molecule going to rotate? And what orientation will it stabilize at? Draw it below por favor.
Gonna rotate clockwise.



(c) *Now* what is the net force on the molecule? You might want to recall the results of calculations in previous homework assignments. Note this is one way the electric force can attract even overall neutral molecules, like when a charged comb attracts bits of neutral paper, or when neutral molecules attract each other into stable solid or liquid configurations. Should get an answer ~ 10^{-33}N.
So force is given by:

$$\begin{aligned}
\mathbf{F} &= q_{Cl}\mathbf{E} + q_H\mathbf{E} \\
&= (-e)\frac{0.0023}{5-l/2}(-\hat{\mathbf{i}}) + e\frac{0.0023}{5+l/2}(-\hat{\mathbf{i}}) \qquad \text{using } E = \frac{0.0023}{r} \text{ from HW 2} \\
&= 0.0023e\left[\frac{1}{5-l/2} - \frac{1}{5+l/2}\right]\hat{\mathbf{i}} \\
&= 0.0023e\left[\frac{l}{5^2-(l/2)^2}\right]\hat{\mathbf{i}} \\
&\approx 0.0023e\left[\frac{l}{5^2}\right]\hat{\mathbf{i}} = 1.87\times10^{-33}\,\mathrm{N}\hat{\mathbf{i}}
\end{aligned}$$

Problem 5. Instead of doing physics I'm going to watch TV….or watch *the* TV, I mean the *inside* of the TV, specifically the *electrons* inside the TV. Huh. It seems those electrons are being accelerated by a 220V potential difference. They must be going really fast. And later, it seems they're being vertically deflected θ = 20° by those orange electric field line things running between the plates. Are electric fields really orange? Is that why oranges are so sour? I don't know. I don't know anything in this class. Hey did you know that the mass of an electron is 9.11×10^{-31}kg!? That sure is small.



(a) Gee, I bet I know which way those electric field lines are pointing.
They must be pointing up, because that would make the force on the negatively charged electron, down.

(b) I can probably even calculate what the strength of that orange electric field is. Maybe not. But by golly I'm a Mustang, and I'm not going down without a fight! At least I remember from PHY 141 that the angle something is traveling at is $\theta = \tan^{-1}(v_y/v_x)$. If I'd forgotten that, I'd be screwed. Or worse. I wonder if gravity matters? Nah, I'll bet the electric field force is muuuuuuuch greater than the gravitational field force.

Kinematics, yo. First we have to get the speed the electrons acquire by being accelerated through that potential 220V potential difference.

$$\sum W_{n.c.} = \Delta KE + \Delta PE_E$$

$$0 = \left[\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right] + q\Delta V$$

$$0 = \frac{1}{2}mv^2 + (-e)(220)$$

$$v = \sqrt{\frac{2(e)(220)}{9.11\times10^{-31}}} = 8.8\times10^6 \text{ m/s}$$

This will remain the x-component of its velocity as it passes through the plate, since the electric force is purely vertical. It will take

$$t = \frac{5\,\text{cm}}{8.8\times10^6\,\text{m/s}} = 5.6\times10^{-9}\,\text{s}$$

seconds to pass through the plate. And by the time it does, it will have acquired a velocity y-component of:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$v_y = -v_x \tan\theta = -(8.8\times10^6)\tan 20^\circ = -3.2\times10^6 \text{ m/s}$$

And now, to get the field, we do,

$$F_y = ma_y$$

$$qE_y = m\frac{\Delta v_y}{\Delta t}$$

$$E_y = \frac{m}{q}\frac{\Delta v_y}{\Delta t} = \frac{9.11\times10^{-31}}{-1.6\times10^{-19}}\frac{-3.2\times10^6}{5.6\times10^{-9}} = 3250\,\text{N/C}$$

(c) I did it! Maybe. How could I even *create* such a field? I know! I'd hook up a battery to those two plates. The battery would charge the plates to its own potential difference, and then the charges would set up the field. But what potential difference would my battery have to have I wonder?

Easy peasy.

$$\Delta V = -\int_{bottom\ plate}^{top\ plate} \mathbf{E} \cdot d\mathbf{r} = \int_0^{0.04} 1200\hat{\mathbf{j}} \cdot dy\hat{\mathbf{j}}$$

$$= 3250 \int_0^{0.04} dy = 130\text{V}$$